The Elements of Euclid

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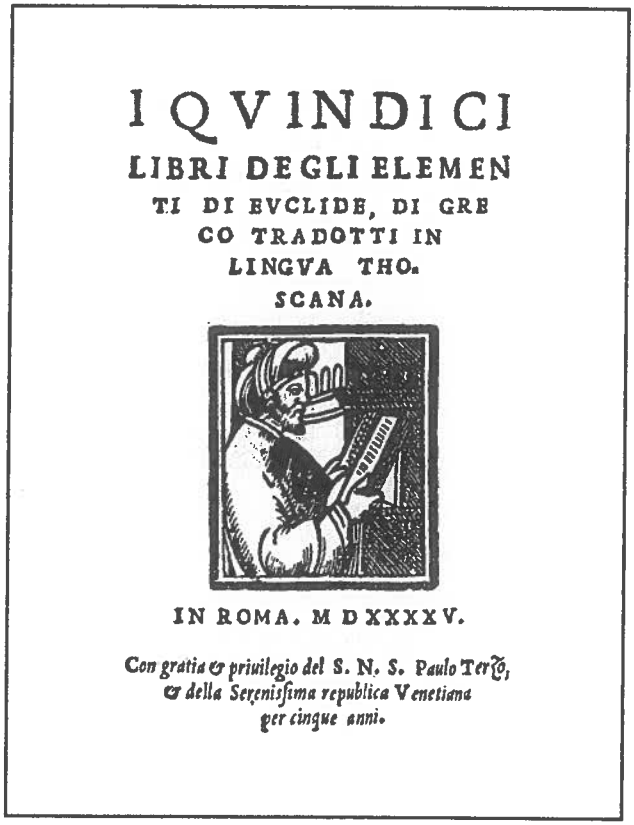
One of the most famous mathematical works in history, **Euclid's *Elements*** (ca 300 B.C.) is devoted to geometry, to algebra, and to number theory. A compilation of his own work and the knowledge of those before him, Euclid's book is of great significance because its contents are presented in a well-organized, logical manner. In his *Elements,* Euclid attempted to develop an organized form of thinking through deductive reasoning. In a deductive system, we establish a few basic statements (postulates) that are accepted as true. We then establish additional statements (theorems) that reflect the logical consequence of either a postulate or some previously established theorem. This process of reasoning, called Euclidean, has penetrated every branch of mathematics.

Euclid's work does have its flaws, though. For instance, some of his definitions don't really "define," because he doesn't base them on a set of primitive (undefined) terms. We see this in his definitions of the terms given below.


I QVINDICI
LIBRI DEGLI ELEMEN
TI DI EVCLIDE, DI GRE
CO TRADOTTI IN
LINGVA THO.
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IN ROMA. M D XXXXV.

Con gratia & priuilegio del S. N. S. Paulo Terzo,
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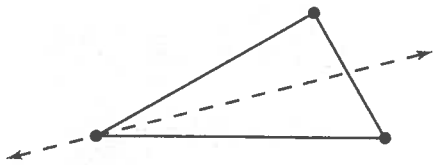
Portrait of Euclid from **I quindici libri degli elementi di Euclide** *(Rome, 1545).*

- **Point**: That which has no part
- **Line**: Breadthless length
- **Straight line**: A line which lies evenly with the points on itself
- **Plane angle**: The inclination to one another of two lines in a plane which meet one another and do not lie in a straight line

Additionally, in some instances Euclid assumed that certain figures held properties that were not logically justified by postulates or previously established theorems. For instance, he assumed that if a line enters a triangle at a vertex, it intersects the opposite side if extended (see figure at right). Today, this assumption is known as Pasch's axiom.



Many of the concepts and ideas in the Elements have been revised and modernized. For example, we now accept *point, line,* and *plane* as primitive terms and use them to create other definitions. Because Euclidean concepts have so heavily influenced the development of geometry, they are still the focus of study in geometry classes around the world. ★

Activities

1. What is logically inconsistent about the idea of defining a point as "the intersection of two lines"?

2. What is Euclid's fifth postulate? Why is it of great historical interest? (For information on Euclid's fifth postulate, see vignette 63.)

3. The Euclidean algorithm, found at the beginning of Book VII of the *Elements,* describes a process for finding the greatest common divisor of two positive integers. Research this algorithm, then use it to find the greatest common divisor of (a) 1,596 and 11,220 and (b) 1,064; 9,338; and 3,003.

4. Show that if p is a prime that divides the product xy, then either p divides x or p divides y.

Related Reading

Adele, Gail. "When Did Euclid Live? An Answer plus a Short History of Geometry." *Mathematics Teacher* (Sept 1989) 460–463.

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Heath, T.L. *History of Greek Mathematics,* Vols I and II. Mineola, NY: Dover, 1981.

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Knorr, W.R. *The Evolution of the Euclidean Elements.* Boston: D. Reidel Publishing, 1975.

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