IB Mathematics Standard Level

Internal Assessment – The Exploration

~ Student Guide ~

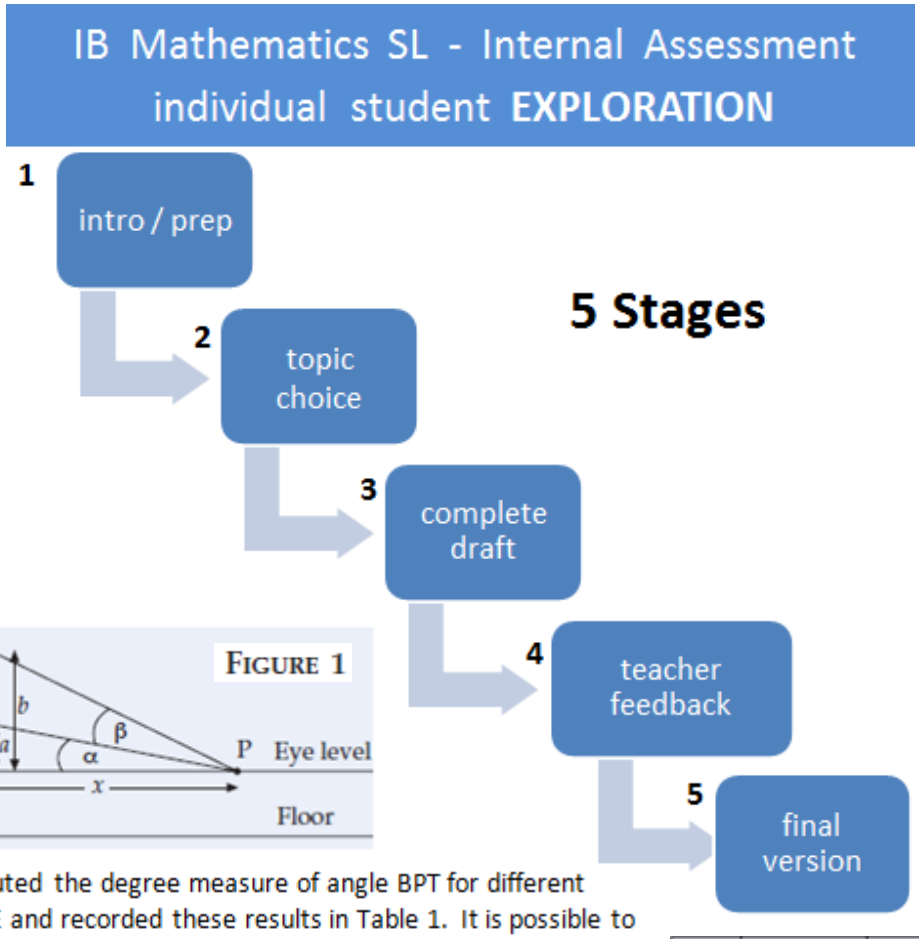


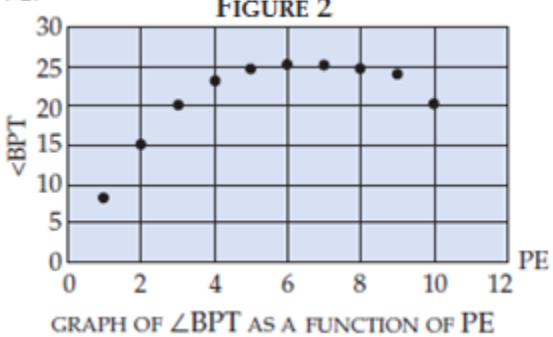
FIGURE 1

I have computed the degree measure of angle BPT for different lengths of PE and recorded these results in Table 1. It is possible to see a relationship between the length of PE and the angle BPT after making a scatterplot of the data as shown in Figure 2. I decided to make PE the independent variable (horizontal axis) and angle BPT the dependent variable (vertical axis) because it gives some insight into how angle BPT changes as a function of PE. I can see from the scatterplot that angle BPT appears to have a maximum value some for a value of PE between PE=5 and PE=8. An important question is whether I can determine an expression for a function that gives angle BPT in terms of PE.

PE	$\tan^{-1}\left(\frac{10}{PE}\right)$	$\tan^{-1}\frac{4}{PE}$	∠BPT
1	84.2895	75.9638	8.3257
2	78.6901	63.4350	15.2551
3	73.3008	53.1301	20.1707
4	68.1986	45.0000	23.1986
5	63.4350	38.6598	24.7752
6	59.0363	33.6901	25.3462
7	55.0080	29.7449	25.2631
8	51.3402	26.5651	24.7752
9	48.0128	23.9625	24.0503
10	41.9872	21.8014	20.1858

TABLE 1 VALUES FOR ∠BPT

FIGURE 2



GRAPH OF ∠BPT AS A FUNCTION OF PE

1. What is Internal Assessment in IB Mathematics Standard Level ?

Internal Assessment (IA) in Maths SL consists of a single internally assessed component (i.e. marked by the teacher) called a mathematical exploration (or just the "Exploration"). The Exploration contributes **20%** to your overall IB score for the course.

2. What is the Exploration ?

Your Exploration is a written report (6-12 pages) involving a mathematical topic that interests you. You will choose a topic in consultation with your teacher after conducting your own research.

3. How is the Exploration assessed ?

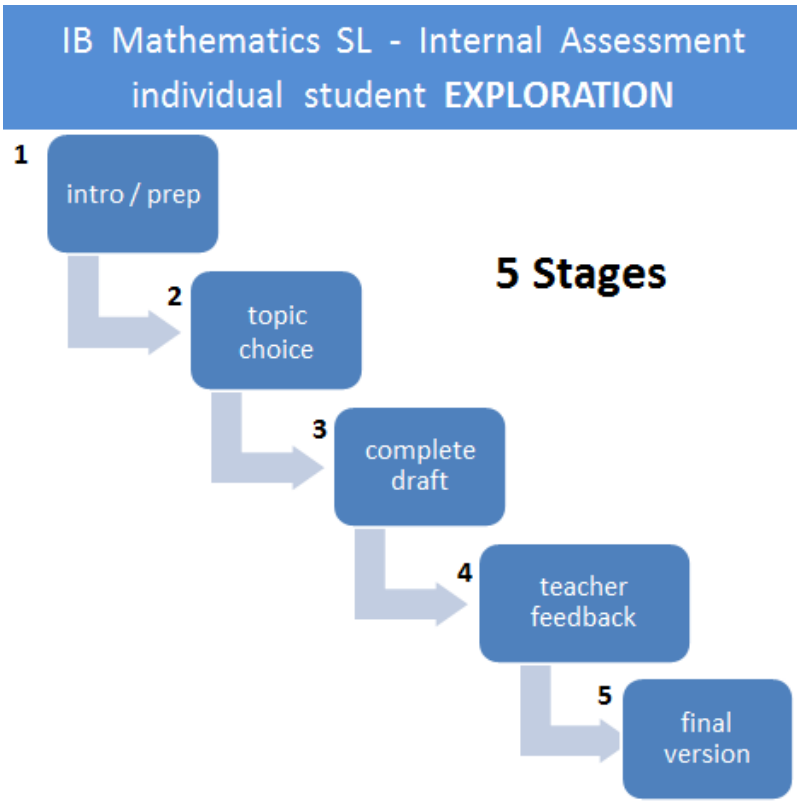
Your Exploration will earn a score out of 20 marks based on the following five criteria. Further details for each criterion and guidance for addressing them is provided later in this guide.

Criterion **A**	max **4** marks	**Communication**
Criterion **B**	max **3** marks	**Mathematical Presentation**
Criterion **C**	max **4** marks	**Personal Engagement**
Criterion **D**	max **3** marks	**Reflection**
Criterion **E**	max **6** marks	**Use of Mathematics**

Some important points to consider:

- In your Exploration you need to write ***about mathematics*** and not just ***do mathematics***.
- Any idea, method, content, etc that is not your own must be **cited** at the point in the Exploration where it is used. Just listing your sources in a bibliography is not enough and may lead to the IB deciding that malpractice has occurred.
- The Exploration is an opportunity for you to learn more about a mathematical topic in which you are genuinely interested. You will be rewarded (**personal engagement**) for explaining your interest in the topic, and for demonstrating curiosity, creativity & independent thinking.
- Your **audience is your fellow students** – that is, you need to write your Exploration so that your classmates in Maths SL can read and understand it. Thus, it is not necessary to explain in great detail basic mathematics that will be familiar to a student in Math SL.
- You will be rewarded (**reflection**) for expressing what you think about the mathematics you are exploring. You should endeavour to pose your own questions and try to answer them using suitably sufficient level of mathematical ideas and procedures.
- You will be required to submit a **complete draft** of your Exploration – containing an introduction, conclusion and all planned content to sufficiently address all five criteria. You will receive feedback on the draft and then be given an opportunity to revise it to submit a final version.
- All of the work you do on your Exploration must be your own. When finished with your final version you will be required to sign a 'declaration' that states, "*I confirm that this work is my own and is the final version. I have acknowledged each use of the words or ideas of another person, whether written, oral or visual*."

Maths SL Exploration Timeline



Stage	Start date	End date **Deadline**
1. Introduction / Preparation		
2. Topic Choice		
3. Writing Draft		
4. Teacher Feedback (written)		
5. Final Version		

Notes:

Step 1 Introduction / Preparation

Read the following two articles. The articles are **not** examples of IA Explorations but appeared in a professional journal for American math teachers and describe in detail how teachers might engage students in the exploration of a particular mathematical problem. Both articles illustrate good writing about mathematics at a level appropriate for SL Maths.

- article 1: ***Pathways and Barriers to Counting***
  https://www.parabola.unsw.edu.au/files/articles/2000-2009/volume-37-2001/issue-1/vol37_no1_3.pdf
- article 2: ***Chasing Imaginary Triangles***
  https://cms.math.ca/crux/v31/n7/page453-456.pdf

Step 2 Choosing a topic for your Exploration

Listed on this page and the next page are 200 possible Exploration topics. Browse through the list and do some very quick research (perhaps 5 min spent looking at a Wikipedia page) on any topic that catches your interest. A quick look at some information about one of the topics may reveal some other topic (not on the list) which interests you. You will be given two weeks to organize a 'short list' of topics (3 to 5) that you will share with your teacher. You will need to consult with your teacher about any potential topic – regarding three important questions: (1) does the topic involve math at a suitable level for an SL Exploration? ; (2) is the topic narrow enough so that it can be treated sufficiently in a 6-12 page report? ; and (3) does the topic lend itself to demonstrating personal engagement (criterion C)? That is, can you envision some way that you could apply something of your own – your own viewpoint, your own examples, your own models (conceptual or physical), your own questions & ideas, etc. ■ Your topic must be approved by your teacher by the given deadline ■

◆ 200 Exploration ideas/topics ◆

Algebra & Number Theory

Modular arithmetic
Chinese remainder theorem
Applications of complex numbers
Diophantine equations
General solution of a cubic equation
Odd perfect numbers
Patterns in Pascal's triangle
Factorable integers of the form *ak* + *b*
Pythagorean triples
Combinatorics – art of counting
Loci and complex numbers
Divisibility tests
Graphical representation of roots of complex numbers
Euler's identity: $e^{i\pi}+1=0$
Probabilistic number theory
Natural logarithms + complex numbers
Twin primes problem
Diophantine application: Cole numbers
Polar equations
Palindrome numbers
Random numbers
Inequalities & Fibonacci numbers
Magic squares & cubes
Roots of unity
Complex numbers & transformations
Goldbach's conjecture
Fermat's last theorem
Continued fractions
Hypercomplex numbers
Applications of logarithms
Euclidean algorithm for GCF
Finding prime numbers
Algebraic congruences
Mersenne primes
Boolean algebra
Matrices & Cramer's rule
Egyptian fractions

Calculus/Analysis & Functions

Mean Value theorem
Applications of power series
Fundamental theorem of calculus
Second order differential equations
Torricelli's trumpet (Gabriel's horn)
Newton's law of cooling
Brachistochrone (min.time) problem
l'Hopital's rule and evaluating limits
Integrating to infinity
Hyperbolic functions
The harmonic series
Torus – solid of revolution

Probability & Probability Distributions

Normal distribution and natural phenomena
Monte Carlo simulations
Insurance and calculating risks
Determination of π by probability
Bayes' theorem
The Monty Hall problem
Random walks
Poisson distribution and queues
Lotteries
The birthday paradox

Geometry

Non-Euclidean geometries
Ptolemy's theorem
Geodesic domes
Minimal surfaces & soap bubbles
Tiling the plane – tessellations
Symmetries of spider webs
Fermat point - polygons & polyhedral
Properties of a regular pentagon
Geometry of the catenary curve
Eratosthenes' - earth's circumference
Constructing a cone from a circle
Mandelbrot set and fractal shapes
Squaring the circle
Architecture and trigonometry
Cavalieri's principle
Hexaflexagons
Proofs of Pythagorean theorem
Map projections
Morley's theorem
Fractal tilings
Pick's theorem & lattices
Nine-point circle
Euler's formula for polyhedral
Ceva's theorem for triangles
Conic sections as loci of points
Curves of constant width
Polyominoes
Spherical geometry
Packing 2D and 3D shapes
Heron's formula
Tesseract – a 4D cube
Penrose tiles
Cycloid curve
Euler line of a triangle
Conic sections
Regular polyhedral
Stacking cannon balls
Area of an ellipse
Consecutive integral triangles
Sierpinksi triangle
Reuleaux triangle

Statistics & Modelling

Logistic function & constrained growth
Modelling epidemics/spread of a virus
Modelling the shape of a bird's egg
Modelling growth of computer power
Modelling change in record performances for a sport
Modelling growth of tumours
Correlation coefficients
Central limit theorem
Least squares regression
Traffic flow
Hypothesis testing
Modelling radioactive decay
Regression to the mean

Numerical Analysis

Methods for solving differential eqns
Methods of approximating π
Estimating size of large crowds
Linear programming
Applications of iteration
Generating the number *e*
Fixed point iteration
Newton's method
Descartes' rule of signs

Logic & Sets

Codes and ciphers
Mathematical induction (strong)
Set theory and different 'size' infinities
Proof by contradiction
Proving a number is irrational

Topology & Networks

Knots
Travelling Salesman Problem
Möbius strip
Steiner problem
Königsberg bridge problem
Klein bottle
Chinese postman problem
Handshake problem

Games & Game Theory

The prisoner's dilemma
Sudoku
Gambler's fallacy
Card games
Knight's tour in chess

Physical, Biological & Social Sciences

Radiocarbon dating
Mathematical methods in economics
Computing centres of mass
Logarithmic scales-decibel, Richter, etc
Fibonacci sequence and spirals in nature
Gravity, orbits & escape velocity
Genetics
Elliptical orbits
Change in BMI for a person over time
Concepts of equilibrium in economics
Biostatistics
Crystallography
Predicting an eclipse

Miscellaneous

Paper folding
Methods of approximating π
Curry's paradox – 'missing' square
Music – notes, pitches, scales, etc
Flatland by Edwin Abbott (book)
Sundials
The abacus
Different number systems
Global positioning system (GPS)
Designing bridges
Barcodes
Voting systems
Towers of Hanoi puzzle
Art of M.C. Escher
Navigational systems
Construction of calendars
Mathematics of juggling
Mathematical card tricks
Applications of parabolas
Terminal velocity
Photography
Harmonic mean
A Beautiful Mind (film)
Slide rules
Airline routes

Step 3 Write a complete draft

Before you start writing your Exploration be sure to carefully read through the details for all five of the assessment criteria that is at the very end of this guide. Along with a brief description and achievement level descriptors, there is also helpful guidance notes for each criterion.

A draft is not an abbreviated or incomplete version of your Exploration. It must be **complete** – including an introduction, a conclusion and a bibliography – with sufficient content to address your stated objective(s) and be in the range of 6 to 12 pages (spacing 1½, font Times New Roman). Your Exploration needs to be logically organized; use appropriate mathematical terminology and notation; include explanatory diagrams, graphs, tables, etc; contain citations to indicate where a source is used; and focuses on the relevant mathematics. It is important to include your own thoughts, questions, reflections & ideas when possible. Write in the first person, e.g. "I decided that the best method is _____ because I realized that …"

Although the Exploration is an individual assignment and all the work must be your own, you are strongly encouraged to regularly consult with your teacher. Your teacher can provide verbal guidance and feedback while you are writing your draft.

■ Submit a paper and electronic version of your draft to your teacher by the given deadline ■

Step 4 Teacher feedback

Your teacher will provide written feedback on the draft of your Exploration. Be sure to ask questions about any comments / feedback that you do not completely understand.

Step 5 Submit final version of your Exploration

From the time you receive written feedback on your draft you will have 6 weeks (3 school weeks & 3 weeks of the winter holiday) to revise your draft and complete the final version of your Exploration. Before submitting your final version complete the **student checklist** on the next page →

■ Submit a paper & electronic version of your final Exploration to your teacher by the deadline ■

See **"The Exploration – Top Tips"** at the very end of this Student Guide. It lists five important points that will help you with your Exploration.

It is absolutely critical that you are completely familiar with all five of the **assessment criteria**. All of the details for the assessment criteria appear on pages 8-10 in this Guide. Carefully read the **Descriptors** and **Further Guidance** for all of the five criteria. Ask your teacher if you have questions or need further clarification.

Mathematics SL Exploration ☑ Student Checklist

Student: _____ date: _____

1. Is your report written entirely by yourself – and trying to avoid simply replicating work and ideas from sources you found during your research? ☐ Yes ☐ No

2. Have you strived to apply your personal interest; develop your own ideas; and use critical thinking skills during your exploration and demonstrate these in your report? ☐ Yes ☐ No

3. Have you referred to the five assessment criteria while writing your report? ☐ Yes ☐ No

4. Does your report focus on good mathematical communication – and read like an article for a mathematical journal? ☐ Yes ☐ No

5. Does your report have a clearly identified introduction and conclusion? ☐ Yes ☐ No

6. Have you documented all of your source material in a detailed bibliography in line with the IB academic honesty policy? ☐ Yes ☐ No

7. Not including the bibliography, is your report 6 to 12 pages? ☐ Yes ☐ No

8. Are graphs, tables and diagrams sufficiently described and labelled? ☐ Yes ☐ No

9. To the best of your knowledge, have you used and demonstrated mathematics that is at the same level, or above, of that studied in IB Mathematics SL? ☐ Yes ☐ No

10. Have you attempted to discuss mathematical ideas, and use mathematics, with a sufficient level of knowledge and understanding? ☐ Yes ☐ No

11. Are formulae, graphs, tables and diagrams in the main body of text? (preferably no full-page graphs; and no separate appendices) ☐ Yes ☐ No

12. Have you used technology – such as a GDC, spreadsheet, mathematics software, drawing & word-processing software – to enhance mathematical communication? ☐ Yes ☐ No

13. Have you used appropriate mathematical language (notation, symbols, terminology) and defined key terms? ☐ Yes ☐ No

14. Is the mathematics in your report performed precisely and accurately? ☐ Yes ☐ No

15. Has calculator/computer notation and terminology **not** been used? ☐ Yes ☐ No
($y = x^2$, not $y = x \wedge 2$; $\approx$, not $=$ for approx. values; π , not pi; $|x|$, not abs(x); etc)

16. At suitable places in your report – especially in the conclusion – have you included reflective and explanatory comments about the mathematical topic being explored? ☐ Yes ☐ No

Criteria for SL Exploration (IA)

Criterion A: Communication (4 marks)

This criterion assesses the organization and coherence of the exploration. A well-organised exploration has an **introduction**, a **rationale** (a brief explanation of why the topic was chosen), describes the **aim** of the exploration and has a **conclusion**. A coherent exploration is logically developed and easy to follow.

Achievement Level	Descriptor
0	The Exploration does not reach the standard described by the descriptors below.
1	The Exploration has some coherence.
2	The Exploration has some coherence and shows some organization.
3	The Exploration is coherent and well organized.
4	The Exploration is coherent, well organized, concise and complete.

◘ Further Guidance ◘

- A **complete** exploration will have all steps clearly explained, and will meet its aim.
- **Organization** refers to the overall structure or framework, including the introduction, body, conclusion etc.
- **A coherent** exploration displays a logical development and is not difficult to follow ('reads well').
- **A concise** exploration remains focused on the overall aim and avoids irrelevant material.
- Key ideas and concepts need to be clearly explained.
- Graphs, tables and diagrams should be embedded in the text where most appropriate and not be put in an appendix at the end of the document.
- The use of technology is not required but strongly encouraged where appropriate.
- It is absolutely critical that the use of a source is cited (footnoted) at the location where it is used.
- Your bibliography must list all sources (books, websites, etc) you consulted when writing your Exploration.

Criterion B: Mathematical Presentation (3 marks)

This criterion assesses to what extent you are able to clearly and effectively use multiple forms of mathematical representation such as formulae, diagrams, tables, graphs and models.

Achievement Level	Descriptor
0	The Exploration does not reach the standard described by the descriptors below.
1	There is some appropriate mathematical presentation.
2	The mathematical presentation is mostly appropriate.
3	The mathematical presentation is appropriate throughout.

◘ Further Guidance ◘

- You are expected to use mathematical language (notation, symbols & terminology) when communicating mathematical ideas, reasoning and findings.
- You should use appropriate technology such as graphic display calculators; and software such as equation editors, spreadsheets, dynamic geometry, computer algebra, drawing and word-processing software along with other mathematical software to enhance the presentation of mathematics in your Exploration.
- The meaning of key terms should clear and any variables or parameters should be explicitly defined.
- All graphs, tables & diagrams should be clearly labelled – and include captions where appropriate.
- Do not use calculator or computer notation unless it is software generated and cannot be changed.

Criterion C: Personal Engagement (4 marks)

This criterion assesses the extent you engage with the exploration and make it your own. This includes independent thinking, creativity, addressing personal interest and presenting math ideas in your own way.

Achievement Level	Descriptor
0	The Exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial personal engagement.
2	There is evidence of some personal engagement.
3	There is evidence of significant personal engagement.
4	There is abundant evidence of outstanding personal engagement.

◘ Further Guidance ◘

- It is important to choose a topic in which you are genuinely interested.
- If it is necessary to include mathematical work from a source such as a textbook in your Exploration then you should endeavour to insert your own comments and description of the work as much as possible.
- Ways to show personal engagement include: investigating your own questions & conjectures; making up your own examples; presenting ideas & results in your own words; creating your own models or functions.

Criterion D: Reflection (3 marks)

This criterion assesses how well you review, analyze and evaluate your exploration. Although reflection may be seen in the conclusion, it should also exist throughout the exploration. Reflection may be demonstrated by considering limitations or extensions, and relating mathematical ideas to your own previous knowledge.

Achievement Level	Descriptor
0	The Exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

◘ Further Guidance ◘

- Simply describing results represents **limited or superficial reflection**. To achieve a score higher than 1 you will need to provide deeper and more sophisticated consideration of methods and results.
- Ways of showing **meaningful reflection** include: linking results to the aim of your Exploration; commenting on what you have learned; considering limitations; or comparing different mathematical approaches.
- Ways of showing **critical reflection** include: considering implications of results; discussing strengths and weaknesses of methods; considering different perspectives; making links between different areas of math.
- **Substantial evidence** is likely to mean that reflection is present throughout the exploration.

Criterion E: Use of Mathematics (6 marks)

This criterion assesses to what extent you use mathematics in your exploration. The mathematics explored should either be part of the syllabus, or at a similar level, or beyond. It should not be completely based on mathematics listed in the prior learning topics. If the level of mathematics is not commensurate with the course, a maximum of two marks can be awarded for this criterion. A piece of mathematics can be regarded as correct even if there are a few minor errors so long as they do not cause a disruption to the flow of mathematics or lead to an incorrect or inaccurate result.

Achievement Level	Descriptor
0	The Exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

◘ Further Guidance ◘

- It is critical that you clearly demonstrate you understand the mathematical concepts and methods you write about in your Exploration. Obtaining a correct answer is not sufficient to 'demonstrate' understanding.
- If only a minimal amount of mathematics commensurate with the SL course is used but the math is central to the development of the exploration the exploration may achieve level 3 or more.
- Regression using technology is commensurate with the SL course but understanding must be demonstrated in order to achieve higher than level 1.
- If the mathematics used is **relevant** to the topic, **commensurate** with the SL course, and clear **understanding** is demonstrated, then the Exploration can achieve a high level in this criterion.

The Exploration – Top Tips

1. Choose a topic in consultation with your teacher that: (i) you're interested in, (ii) involves math at a level suitable for Math SL, (iii) is narrow enough for 6-12 pages, (iv) has opportunities for personal engagement.

2. Your Exploration must have an aim or objective which involves doing some mathematics. It is important to maintain a focus on the overall aim/objective and a focus on mathematical concepts and methods.

3. Although all the work on your Exploration must be your own, do not hesitate to ask your teacher for advice and feedback at any stage. Your teacher will provide written feedback on your draft.

4. Be sure you fully understand the expectations of the five assessment criteria, and refer back to them while you are planning and writing your Exploration.

5. The Exploration is an opportunity to complete a significant assessment item (20% of IB score) while <u>not</u> under the pressure of timed exam conditions. Take advantage of the opportunity by following instructions, meeting deadlines, engaging & reflecting in your own way, and enjoying some math you are interested in.